figuration, and g a function of temperature defined by

$$g = \rho_0 \int_{T_0}^{T} dT' \int_{T_0}^{T'} (C/T'')\, dT'', \tag{2}$$

where C is the specific heat.

From its general definition the free energy satisfies the relation

$$F = E - TS, \tag{3}$$

where E is internal energy and S is entropy. Denoting time derivatives by means of superposed dots, differentiation of (3) yields

$$\dot{F} = \dot{E} - \dot{T}S - T\dot{S}. \tag{4}$$

The energy balance (or First Law) can be written as

$$\dot{Q} = \dot{E} - \sigma_i \dot{\varepsilon}_i/\rho. \tag{5}$$

We assume that each element of material responds adiabatically. This means principally that we neglect heat conduction, a reasonable approximation in the context of the wave propagation problem under consideration. Thus, the rate of heat flow, $\dot{Q}$, is zero and

$$\dot{E} = \sigma_i \dot{\varepsilon}_i/\rho. \tag{6}$$

The free energy balance then becomes

$$\dot{F} = -\dot{T}S - T\dot{S} + \sigma_i(\dot{\varepsilon}_i^e + \dot{\varepsilon}_i^p)/\rho. \tag{7}$$

Here, the rate-of-work of the stresses has been separated into elastic and plastic components simply under the formulation of (KG). The elastic work term can be decomposed further into dilatational and deviatoric contributions to obtain

$$\dot{F} = -\dot{T}S - T\dot{S} + (\sigma_i + p)\dot{e}_i^e/\rho - p\dot{e}^e/\rho + \sigma_i \dot{\varepsilon}_i^p/\rho. \tag{8}$$

An equivalent expression for F is obtained by differentiating (1) with respect to time, making use of the chain rule and the definition (2):

$$\dot{F} = Ke^e\dot{e}^e/\rho_0 + 2Ge_i^e\dot{e}_i^e/\rho_0 - K\beta(T-T_0)\dot{e}^e/\rho_0 - K\beta e^e\dot{T}/\rho_0 - \dot{T}\int_{T_0}^{T} (C/T')\, dT'. \tag{9}$$

Comparison of coefficients of the independent variables e, e_i^e, $\dot{T}$, etc. between (8) and (9) shows that

$$p/K = -(\rho/\rho_0)[e^e - \beta(T-T_0)], \tag{10}$$

$$(\sigma_i + p)/2G = (\rho/\rho_0)e_i^e, \tag{11}$$

$$\rho_0 S = K\beta e^e + \rho_0 \int_{T_0}^{T} (C/T')\, dT', \tag{12}$$

$$\rho_0 T\dot{S} = (\rho_0/\rho)\sigma_i \dot{\varepsilon}_i^p. \tag{13}$$

These constitutive equations must be considered as approximations, and it is possible that for some materials (10) may be inadequate for large compression. The results do not depend in an essential way in the choice of pressure–volume relation and replacement of (10) by an empirical relation more suitable for given experimental data should be readily accommodated. Equation (13) is of particular interest since it indicates that entropy production, required to be positively the Clausius–Duhem inequality, is due entirely to plastic work.

To determine the temperature at a point at any time it is necessary to know the stress and strain histories up to that time and these will depend in general on the temperature history. It is clear from (12) and (13) that the temperature is given by the integration of

$$K\beta T\dot{\varepsilon}^e + \rho_0 C\dot{T} = (\rho_0/\rho)\sigma_i\dot{\varepsilon}_i^p. \tag{14}$$

In the circumstances which pertain to the propagation of a plane wave certain simplifications occur. Isochoric plastic deformation, symmetry and kinematics require that

$$\left.\begin{array}{c}\varepsilon_1^p + \varepsilon_2^p + \varepsilon_3^p = 0, \qquad \varepsilon_2 = \varepsilon_2 = 0, \qquad \varepsilon_1 = e = \ln(\rho_0/\rho), \\ \varepsilon_2^p = \varepsilon_3^p = -\varepsilon_{1/2}^p, \qquad \varepsilon_2^e = \varepsilon_3^e = \varepsilon_{1/2}^p.\end{array}\right\} \tag{15}$$

We need, therefore, only consider one strain component ε_1 and its elastic and plastic parts, the remaining components being obtained if needed from (15). Similarly, we need only consider the stress σ_1 and the pressure p, the transverse stresses $\sigma_2 = \sigma_3$ being obtained from $\sigma_1 + 2\sigma_2 = -3p$. Thus, we now drop the subscripts 1, 2, 3 and use σ to refer to σ_1 and ε to refer to ε_1.

Under conditions of uniaxial strain the plastic work takes the form

$$(\rho_0/\rho)\sigma_i\dot{\varepsilon}_i^p = \tfrac{3}{2}(\rho_0/\rho)(\sigma + p)\dot{\varepsilon}^p. \tag{16}$$

Furthermore, it is easy to show through (15) and (11) that the plastic strain can be written as

$$\varepsilon^p = 2\varepsilon/3 - (\rho_0/\rho)(\sigma + p)/2G. \tag{17}$$

Using the above relations, (14) for the temperature variation becomes

$$\rho_0 C\dot{T} = -K\beta T\dot{\varepsilon} + (\rho_0/\rho)(\sigma + p)\dot{\varepsilon} - (3/8G)[\{(\rho_0/\rho)(\sigma + p)\}^2]^{\cdot} \tag{18}$$

Here, the symbol $[\ ]^{\cdot}$ denotes derivative with respect to time of the enclosed quantity.

We note first that if the plastic work is entirely neglected and the specific heat is assumed constant the temperature is given by

$$T = T_0(\rho/\rho_0)^{\mu}, \tag{19}$$

where $\mu = K\beta/\rho_0 C$. The temperature rise is more generally given by the solution of the non-linear integral equation

$$\rho_0 \int_{T_h}^{T} C\, dT = -K\beta \int_{\varepsilon_h}^{\varepsilon} T\, d\varepsilon + \int_{\varepsilon_h}^{\varepsilon} (\rho_0/\rho)(\sigma + p)\, d\varepsilon$$
$$-(3/8G)\{(\rho_0/\rho)(\sigma + p)\}^2 + (3/8G)\{(\rho_0/\rho_h)(\sigma_h + p_h)\}^2. \tag{20}$$

Here, the subscript h refers to the Hugoniot elastic limit, i.e. the conditions at the precursor wavefront. Since the precursor is conventionally treated as an elastic discontinuity, no plastic deformation occurs, and the deviatoric stress and temperature at the wavefront are, therefore, related to the density change through (11) and (19). These give

$$(\rho_0/\rho_h)(\sigma_h + p_h) = 2G\ln(\rho_0/\rho_h), \tag{21}$$

$$T_h = T_0(\rho_h/\rho_0)^{\mu}. \tag{22}$$

In order to find an approximate solution to (20), we first note that the second term on the right-hand side of the equation is the total work of the deviatoric stress while the third term is that fraction of the work that is elastically recoverable. Comparison of the second term with the first requires that the deviatoric stress integrand be compared to a fictitious stress of magnitude βT times the bulk modulus. For common metals at room temperature, βT is a fraction in the range 0·004–0·025, and this fraction of the bulk modulus will be larger than the deviatoric stress except for very strong shocks in materials having the smallest values of β [see (KG, Fig. 3)]. If the